This research was performed partly under the auspices of the U.S. Atomic Energy Commission, and partly with the support of the National Research Council of Canada. K.A.K. and J.P.A. are particularly grateful to Dr G. Friedlander for allowing use of the neutron diffraction facilities at Brookhaven National Laboratory and to S.J. La Placa for technical assistance during the experiment. Major computer programs used in this study were written by A. C. Larson and J. S. Stephens and modified by the authors to meet the constraints of the local computer installation.

## References

Allen, H. C. \& Plyler, E. K. (1956). J. Chem. Phys. 25, 1132-1136.
Beaucage, D. R., Kelley, M. A., Ophir, D., Rankowitz, S., Spinrad, R. J. \& van Norton, R. (1966). Nucl. Instr. Methods, 40, 26-44.
Bondi, A. (1964). J. Phys. Chem. 68, 441-451.

Brown, I. D. \& Shannon, R. D. (1973). Acta Cryst. A 29, 266-282.
Hamilton, W. C. (1965). Acta Cryst. 18, 502-510.
Hamilton, W. C. \& Ibers, J. A. (1968). Hydrogen Bonding in Solids. New York: W. A. Benjamin.
International Tables for X-ray Crystallography (1968). Vol. III, p. 197. Birmingham: Kynoch Press.
IUPAC-IUB Commission on Biochemical Nomenclature (1970). J. Mol. Biol. 52, 1-17.

Kerr, K. A. \& Ashmore, J. P. (1973). Acta Cryst. B29, 2124-2127.
Lehmann, M. S. \& Larsen, F. K. (1974). Acta Cryst. A30, 580-584.
Neutron Diffraction Commission (1969). Acta Cryst. A 25, 391-392.
Paul, I. C. (1974). In Chemistry of the Thiol Group, edited by S. Patai: New York. John Wiley.
Srinivasan, R. \& Сhacko, K. K. (1967). In Conformation of Biopolymers, edited by G. N. Ramachandran, Vol. I, pp. 61-68. London, New York: Academic Press.
Young, R. A. (1969). Acta Cryst. A 25, 55-66.
Zachariasen, W. H. (1963). Acta Cryst. 16, 1139-1144.

# The Crystal Structure of Hopeite, $\mathbf{Z n}_{\mathbf{3}}\left(\mathbf{P O}_{\mathbf{4}}\right)_{2} \mathbf{4} \mathbf{H}_{\mathbf{2}} \mathbf{O}$ 

By A. Whitaker<br>Department of Physics, Brunel University, Kingston Lane, Uxbridge, Middlesex, England

(Received 13 January 1975; accepted 18 February 1975)


#### Abstract

The structure of hopeite, $\mathrm{Zn}_{3}\left(\mathrm{PO}_{4}\right)_{2} .4 \mathrm{H}_{2} \mathrm{O}$, has been redetermined. It crystallizes in the orthorhombic system with cell dimensions $a=10 \cdot 629$ (2), $b=18 \cdot 339$ (3), $c=5 \cdot 040$ (1) $\AA$. The space group is Pnma and there are four molecules in the unit cell. Least-squares refinement has been completed on three-dimensional data ( 1471 structure factors). The $\mathbf{H}$ atoms have been located and included in the refinement with an isotropic temperature factor of $4 \cdot 0$. The final residual is $6 \cdot 8 \%$. The structure consists of $\mathrm{ZnO}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}$ octahedra, $\mathrm{ZnO}_{4}$ tetrahedra and $\mathrm{PO}_{4}$ tetrahedra, none of which are regular; these polyhedra share corners and edges. The mean (uncorrected) bond lengths are 2.099 (3) $\AA$ in the $\mathrm{ZnO}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}$ octahedron, 1.963 (3) $\AA$ in the $\mathrm{ZnO}_{4}$ tetrahedron and 1.537 (3) $\AA$ in the $\mathrm{PO}_{4}$ tetrahedron. The thermal vibrations have been analysed and the mean corrected bond length in the $\mathrm{ZnO}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}$ octahedron is $2 \cdot 101 \AA$, assuming riding motion.


## Introduction

The crystal structure of hopeite has been determined twice with different results. In one case the structure reported (Mamedov, Gamidov \& Belov, 1961; Gamidov, Golovachev, Mamedov \& Belov, 1963) suggests that hopeite is similar to the diaspore AlOOH -goethite FeOOH structure and the structure factor of the ( $h k 0$ ) projection refined to a residual of $24 \%$ even without corrections for temperature factors. However, using the same projection, Liebau (1965) reported a completely different structure with certain similarities to the already reported structure of phosphophyllite, $\mathrm{Zn}_{2} \mathrm{Fe}\left(\mathrm{PO}_{4}\right)_{2} .4 \mathrm{H}_{2} \mathrm{O}$ (Kleber, Liebau \& Piatkowiak, 1961); in this case a final residual of $14.4 \%$ was given.

In view of the completely different structure, it was decided to redetermine the structure with threedimensional data.

## Experimental

Attempts to grow crystals suitable for investigation failed but finally a naturally occurring specimen was obtained by courtesy of the Keeper of Minerals, Department of Mineralogy, British Museum (Natural History). The original source of the specimen was the Broken Hill Mines, North-Western Rhodesia (specimen number BM 1907, 240). The crystal chosen for the intensity measurements was colourless and of triangular prismatic habit parallel to [001]. The crystal optics and

Laue symmetry confirmed that the crystals belong to the orthorhombic system.

The cell dimensions were determined by a leastsquares fit to 12 strong intensity diffraction beams on a four-circle diffractometer using monochromatic molybdenum radiation (Mo $K \alpha=0.71070 \AA$ ) and are $a=$ 10.629 (2), $b=18.339$ (3), $c=5.040$ (1) $\AA$. These values are in good agreement with values reported previously (Wolfe, 1940; Mamedov et al., 1961; Liebau, 1965).

The calculated density for $Z=4$ is $D_{x}=3.096 \pm 0.001$ $\mathrm{g} \mathrm{cm}{ }^{-3}$; this is in good agreement with the value obtained by de Schulten (1904) on artificial crystals, $3.109 \mathrm{~g} \mathrm{~cm}^{-3}$, and within the range of densities obtained by Spencer (1908), $3 \cdot 0$ to $3 \cdot 1 \mathrm{~g} \mathrm{~cm}^{-3}$, for natural crystals from the same area of Rhodesia as the present specimen.

Systematic absences ( $h k 0$ absent when $h=2 n+1$, $0 k l$ absent when $k+l=2 n+1, h 00$ absent when $h=$ $2 n+1,0 k 0$ absent when $k=2 n+1,00 l$ absent when $l=2 n+1$ ) were obtained from ( $h k 0$ ), ( $h k 1$ ) and ( $h k 2$ ) Weissenberg photographs and ( 0 kl ) and ( h 0 l ) precession photographs. These indicate the space group to be either Pnma (No. 62) or Pn2 ${ }_{1} a$ (No. 33); the former was taken to be correct because of previous work (Gamidov et al., 1963; Liebau, 1965).

Integrated intensity data were collected on an automatic four-circle Stoe diffractometer using monochromatic molybdenum radiation. In this way the intensities of 1415 independent reflexions were measured; to these were added another 56 reflexions with zero intensity and all reflexions were processed. The intensity of a standard reflexion $(0,13,1)$ was measured after every 25 reflexions.

The observed intensities were corrected for absorption (linear absorption coefficient $=77 \mathrm{~cm}^{-1}$ ) and Lo-rentz-polarization factors.

## Determination of the structure

The structure was solved, independently of previously reported structures, from a three-dimensional Patterson synthesis using 1276 reflexions of intensity greater than three times the appropriate standard deviation, and partially phased Fourier syntheses. The Fourier syntheses produced two sets of coordinates for the structure, designated S1 and S2.

These two structures are related to each other (Fig. 1) and the $x$ and $y$ coordinates in the second of these (S2) are in good agreement with those given by Gamidov et al. (1963). The projection is hence the same but

Table 1. Parameters and standard deviations after anisotropic refinement
The anisotropic temperature factor is defined as $\exp \left\{-\left(B_{11} h^{2}+B_{22} k^{2}+B_{33} l^{2}+2 B_{12} h k+2 B_{13} h l+2 B_{23} k l\right) \times 10^{-5}\right\}$.

|  | $x$ | $y$ | $z$ | $B_{11}$ or $B$ | $B_{22}$ | $B_{33}$ | $B_{12}$ | $B_{13}$ | $B_{23}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zn (1) | $0 \cdot 23618$ (13) | $0 \cdot 25000$ (0) | $0 \cdot 57284$ (29) | 215 (10) | 144 (4) | 1151 (50) | 0 (0) | 42 (18) | 0 (0) |
| $\mathrm{Zn}(2)$ | $0 \cdot 14267$ (7) | -0.00087 (5) | $0 \cdot 20768$ (16) | 77 (6) | 70 (2) | 393 (29) | 0 (3) | 1 (10) | - 15 (6) |
| P | $0 \cdot 10285$ (16) | 0.09421 (4) | 0.72589 (35) | 133 (12) | 43 (4) | 425 (56) | 8 (6) | 17 (21) | -19 (13) |
| O(1) | $0 \cdot 10737$ (75) | $0 \cdot 25000$ (0) | $0 \cdot 25730$ (169) | 214 (59) | 190 (26) | 760 (260) | 0 (0) | - 162 (109) | 0 (0) |
| $\mathrm{O}(2)$ | $0 \cdot 38614$ (73) | $0 \cdot 25000$ (0) | $0 \cdot 84901$ (183) | 92 (55) | 244 (30) | 1550 (323) | 0 (0) | -191(115) | 0 (0) |
| $\mathrm{O}(3)$ | $0 \cdot 33521$ (57) | $0 \cdot 16977$ (35) | $0 \cdot 33917$ (133) | 295 (46) | 143 (17) | 1971 (243) | 80 (24) | 99 (91) | 110 (55) |
| $\mathrm{O}(4)$ | $0 \cdot 14010$ (56) | $0 \cdot 17265$ (30) | $0 \cdot 78465$ (114) | 413 (48) | 80 (14) | 993 (200) | -51 (22) | 318 (83) | -68 (44) |
| $\mathrm{O}(5)$ | $0 \cdot 10004$ (69) | 0.07994 (33) | 0.42983 (108) | 822 (69) | 106 (16) | 268 (175) | 132 (28) | -230 (91) | - 59 (45) |
| $\mathrm{O}(6)$ | $0 \cdot 47552$ (46) | 0.07867 (29) | $0 \cdot 64140$ (123) | 54 (34) | 78 (14) | 2113 (228) | -17 (18) | -91(76) | 123 (48) |
| $\mathrm{O}(7)$ | $0 \cdot 19796$ (45) | $0 \cdot 04067$ (28) | 0.86067 (103) | 98 (34) | 88 (14) | 724 (173) | 41 (18) | 186 (66) | 71 (41) |
| H(14) | $0 \cdot 122$ (21) | $0 \cdot 188$ (12) | $0 \cdot 121$ (46) | $4 \cdot 0$ |  |  |  |  |  |
| H(24) | 0.456 (21) | $0 \cdot 198$ (11) | $0 \cdot 792$ (48) | $4 \cdot 0$ |  |  |  |  |  |
| H(35) | 0.233 (21) | $0 \cdot 134$ (12) | $0 \cdot 248$ (50) | $4 \cdot 0$ |  |  |  |  |  |
| H(36) | $0 \cdot 396$ (21) | $0 \cdot 136$ (12) | $0 \cdot 493$ (48) | $4 \cdot 0$ |  |  |  |  |  |



Fig. 1. The $c$-axis projections of three proposed structures of hopeite, (a) that designated $\mathrm{S} 1,(b)$ that designated S 2 and (c) that proposed by Liebau (1965).
their $z$ coordinates obtained from packing considerations, are not the same. Fig. $1(c)$ is the ( 001 ) projection of the structure given by Liebau (1965). Again there are similarities between this structure and the other two, the main difference being a movement in the position of the atoms with respect to the origin. Incidentally there is a mistake in Liebau's Fig. 1; to agree with his coordinates the origin of this figure should be at the top left-hand corner instead of the bottom left-hand corner; this also transposes the diagram into a righthand set of axes.
Each of these sets of coordinates received four cycles of least-squares refinement using individual isotropic temperature factors, and a weighting scheme such that $\omega=1 / \sigma^{2}$ (where $\sigma$ is the standard deviation); the residuals of the initial data ( $R_{0}$ ) and after each cycle of refinement ( $R_{1}-R_{4}$ ) were as follows:

|  | $R_{0}$ | $R_{1}$ | $R_{2}$ | $R_{3}$ | $R_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| S1 | $233 \cdot 7 \%$ | $64 \cdot 5 \%$ | $209 \cdot 8 \%$ | $93 \cdot 0 \%$ | $58 \cdot 5 \%$ |
| S2 | $47 \cdot 2$ | $13 \cdot 7$ | $10 \cdot 3$ | $10 \cdot 0$ | $10 \cdot 0$ |

In addition, from the second cycle on, the output for the first crystal structure recommended negative temperature factors for all atoms except $\mathrm{O}(7)$. Because of this and the increased value of the residual the refinement of this structure was discontinued. The atomic scattering factors used for $\mathrm{Zn}, \mathrm{P}, \mathrm{O}$ and later H were those for the neutral atoms (International Tables for X-ray Crystallography, 1962).

At this stage in the investigation it was noticed that repeated measurements of the intensity of the standard reflexion varied by about $\pm 4 \%$ and because of this the weighting scheme was altered so that $\omega=1 /(\sigma+0.025$ $\left.F_{o}\right)^{2}$ where $F_{o}$ is the observed structure factor. The structure was then refined by full-matrix least-squares calculations, the function minimized being $\sum \omega \Delta F^{2}$; $\Delta F=F_{\text {obs }}-F_{\text {calc }}$. The refinement was continued for both isotropic and anisotropic temperature factors until the recommended shifts were less than one-tenth of the appropriate standard deviation (Mason, 1964).

Anisotropic refinement was completed and gave a final residual of $6.8 \%$ including zero intensities and $6.4 \%$ omitting them.

A three-dimensional difference Fourier synthesis using all data was calculated at the end of this refinement, but it was not possible to obtain the H positions from this owing to the large number of spurious peaks. However, the H atoms were obtained from a three-dimensional difference synthesis using only those reflexions for which $\sin \theta / \lambda \leq 1 \cdot 1$ ( 616 reflexions). $\mathbf{H}(14), \mathbf{H}(35)$ and $\mathbf{H}(36)$ were clearly visible but the fourth H could be either $\mathrm{H}(23)$ or $\mathrm{H}(24)$ and each of these positions was used in turn. With the same reflexions, the refinement was continued in stages as follows; first, only the heavy atoms were refined; secondly, the positions of the H atoms were refined alone, with a constant isotropic temperature factor of 4.0 for H and half the recommended shifts ( H atoms are denoted by two numbers,


Fig. 2. A composite electron density map of the asymmetric unit calculated on sections through the atomic peaks. The contouring is at intervals of 12 e $\AA^{-3}$ around the zinc, $8 \mathrm{e} \AA^{-3}$ around the phosphorus and $4 \mathrm{e} \AA^{-3}$ around the other atoms. The outermost contour is at the first interval.


Fig. 3. The $c$-axis projection of the complete cell. Those groupings associated with zinc or phosphorus atoms with $z$ coordinate equal to or greater than 0.5 are drawn with bold lines.
the first is that of the donor oxygen and the second that of the probable receptor oxygen).

The configuration involving $\mathrm{H}(23)$ could not be refined satisfactorily whereas that involving $\mathrm{H}(24)$ could, and so the latter was taken as the most probable position of the fourth H atom. Attempts to refine the temperature factors of the H atoms failed.

Still with the reduced set of data attempts were made to refine simultaneously the positions and temperature factors of the heavy atoms and the positions of the H atoms. This failed for $H(24)$, so the attempt was repeated but this atom was excluded from the refinement. This latter refinement was successful and these coordinates of the H atoms are given in Table 1. It was noticed that the standard deviations of these coordinates increased by about $10 \%$ when attempts were made to simultaneously refine them with the heavyatom coordinate and temperature factors. These increased standard deviations are included in Table 1, for $\mathrm{H}(24)$ as well as the others.

The positions and temperature factors of the heavy atoms were further refined by using the full data until the recommended shifts were again less than one tenth of the appropriate standard deviation (Mason, 1964). The final coordinates and temperature factors for all the atoms are given in Table 1 ; the final residual was $6 \cdot 8 \%$ including zeros, $6 \cdot 5 \%$ omitting them.

A composite electron density map of the asymmetric unit is given in Fig. 2.*

## Discussion of the structure

The structure consists of $\mathrm{ZnO}_{6}$ octahedra, $\mathrm{ZnO}_{4}$ tetrahedra and $\mathrm{PO}_{4}$ tetrahedra, all of which are significantly distorted (see below). The $\mathrm{ZnO}_{6}$ octahedra consist of four $\mathrm{H}_{2} \mathrm{O}$ molecules and two O atoms and thus may be represented as $\mathrm{ZnO}_{2}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}$, the two O atoms in this polyhedron are shared with two $\mathrm{PO}_{4}$ tetrahedra. Each $\mathrm{PO}_{4}$ tetrahedron shares one corner with the $\mathrm{ZnO}_{6}$ octahedron, two other corners with different $\mathrm{ZnO}_{4}$ tetrahedra and an edge with a third. This means that one O atom, $\mathrm{O}(7)$, is involved in three polyhedra, one $\mathrm{PO}_{4}$ and two $\mathrm{ZnO}_{4}$ (this is discussed below). Each $\mathrm{ZnO}_{4}$ tetrahedron shares two corners with two different $\mathrm{PO}_{4}$ tetrahedra, an edge with a third and two corners with different $\mathrm{ZnO}_{4}$ tetrahedra which are in identical positions in adjacentcells. A $c$-axis projection of the complete cell is given in Fig. 3, but to reduce possible confusion the $\mathrm{ZnO}_{4}$ tetrahedron has not been shown as a tetrahedron but as four linkages to the appropriate $O$ atoms.

Comparison of the present structure with that obtained by Liebau (1965) shows that the major differences are firstly that the atoms are all moved with respect to the origin and secondly that the orientation

[^0]of the phosphate tetrahedra with respect to the adjacent $\mathrm{ZnO}_{6}$ octahedron is different. However the present structure is completely different from that of Mamedov et al., (1961). A further check on the possible error of the latter structure was obtained by using their coordinates as the initial data for four cycles of leastsquares refinement. The residuals for the initial data and after four cycles of refinement using isotropic temperature factors were $62.0 \%$ and $58.6 \%$ respectively, thus indicating the essential incorrectness of their structure.

There would appear to be two major structural differences between hopeite and parahopeite (Kumbasar \& Finney 1968; Chao, 1969). Both structures contain $\mathrm{ZnO}_{6}$ octahedra where Zn is coordinated by four $\mathrm{H}_{2} \mathrm{O}$ molecules and two O atoms. In hopeite the two O atoms are at adjacent corners of the octahedron while in parahopeite they are on opposite corners. Secondly both structures contain $\mathrm{ZnO}_{4}$ tetrahedra but in hopeite each of these share two corners with others whereas in parahopeite they are independent.

Palache, Berman \& Frondel (1963) mention that hopeite has three cleavages: $\{010\}$ perfect, $\{100\}$ good and $\{001\}$ poor. Ignoring hydrogen bonds the first of these cleavages may be accounted for by the breaking of four $\mathrm{Zn}(1)-\mathrm{O}$ bonds [two $\mathrm{Zn}(1)-\mathrm{O}(3)$ and two $\mathrm{Zn}(1)-\mathrm{O}(4)]$ per unit cell and the second by the breaking of four $\mathrm{Zn}(2)-\mathrm{O}(6)$ bonds per unit cell. In view of the fact that the latter bonds are shorter than the former, this cleavage would be more difficult to accomplish and it would have to be stepped. The third cleavage requires the breaking of eight bonds per unit cell, four $\mathrm{Zn}(2)-\mathrm{O}(7)$ and four $\mathrm{Zn}(2)-\mathrm{O}(5)$ and would be even more difficult.

## Bond lengths and angles uncorrected for thermal vibration

## $\mathrm{ZnO}_{6}$ octahedron

The individual values of the $\mathrm{Zn}-\mathrm{O}$ and $\mathrm{O}-\mathrm{O}$ distances are given in Table 2. The weighted mean values are: $\mathrm{Zn}-\mathrm{O} 2.099$ (3), O-O 2.955 (3) $\AA$. Detailed examination oi the individual values indicates that the octahedron is very distorted, the range of $\mathrm{Zn}-\mathrm{O}$ distances being $12 \sigma$ where $\sigma$ is the largest individual standard deviation. In addition the $\mathrm{O}-\mathrm{Zn}-\mathrm{O}$ angles vary considerably over the range $84.5^{\circ}$ to $94.0^{\circ}$, this range corresponding to $47 \sigma$. Some of this distortion may be due to thermal vibration but it is thought that, because equivalent values vary so much, the distortion must be due to additional causes.

Previous reported values of this bond length tend to be somewhat scattered (Table 3); the present value is in reasonable agreement with the others. Significant distortion ( $>3 \sigma$ ) occurs in another compound in this table (Stählin \& Oswald, 1970).

Of the four independent O atoms in this group, three will belong to $\mathrm{H}_{2} \mathrm{O}$ molecules. Assuming that there is no hydrogen bonding between atoms within
the same octahedron and that the hydrogen bonds will be directed away from the Zn , then $\mathrm{O}(1), \mathrm{O}(2)$ and $\mathrm{O}(3)$ will be $\mathrm{H}_{2} \mathrm{O}$ molecules. This was confirmed by a difference synthesis. The position of the H atom associated with $\mathrm{O}(1)$ and $\mathrm{O}(3)$ was clearly obtained but that associated with $O(2)$ could be either directed approximately towards $\mathrm{O}(3)$, designated $\mathrm{H}(23)$ or towards $\mathrm{O}\left(4^{i i}\right)$ designated $\mathrm{H}(24)$. The initial position of the former was obtained from the difference map as 0.3845 , $0.1953,0.9139$ by Booth's (1948) method.

Consider now the sum of the angles $\mathrm{Zn}-\mathrm{O}-\mathrm{H}_{a}$, $\mathrm{Zn}-\mathrm{O}-\mathrm{H}_{b}$ and $\mathrm{H}_{a}-\mathrm{O}-\mathrm{H}_{b}$. For $\mathrm{O}(1)$ and $\mathrm{O}(3)$ this is $333(20)^{\circ}$ and $317(21)^{\circ}$ respectively while for $\mathrm{O}(2)$ it is either $346(25)^{\circ}$ or $315(21)^{\circ}$ depending on whether the H atom is $\mathrm{H}(23)$ or $\mathrm{H}(24)$ respectively. The errors involved in these angles are so large that it is not possible to decide whether the water oxygen, the associated hydrogens and the zinc are coplanar.

If one considers the O atoms about each water O atom, then the sums of the angles subtended by these and the Zn atom are $309 \cdot 8(5)^{\circ}, 288 \cdot 0(5)^{\circ}$ for $\mathrm{O}(1)$ and $\mathrm{O}(3)$ respectively and $295 \cdot 2(5)^{\circ}$ or $289 \cdot 6(5)^{\circ}$ for $\mathrm{O}(2)$ depending on whether one considers $\mathrm{O}(3)$ or $\mathrm{O}\left(4^{\mathrm{ii})}\right.$ ) to be the related atom. In either case it would appear that the atomic arrangement of the heavier atoms about the water oxygens is significantly not coplanar and this is different from the case of the $\mathrm{Mg} .\left(\mathrm{H}_{2} \mathrm{O}\right)_{6}$ octahedron, where it is (Baur, 1964; Whitaker \& Jeffery, 1970; Whitaker, 1973). Another point emerges: if the H atom associated with $\mathrm{O}(2)$ is $\mathrm{H}(23)$, could this be involved in a hydrogen bond to $\mathrm{O}(3)$ ? As the configuration about $O(3)$ is not significantly different from that about the
other atoms, it would appear that $\mathrm{H}(23)$, if it exists, does not take part in hydrogen bonding.

## $\mathrm{ZnO}_{4}$ tetrahedron

The individual values of the $\mathrm{Zn}-\mathrm{O}$ and $\mathrm{O}-\mathrm{O}$ distances are given in Table 2. The weighted mean values are: $\mathrm{Zn}-\mathrm{O} 1.963$ (3), O-O $3 \cdot 189$ (3) $\AA$. Detailed examination of the individual values indicates that the tetrahedron is very distorted, the range of $\mathrm{Zn}-\mathrm{O}$ distances being $14 \sigma$ where $\sigma$ is the largest individual standard deviation. In addition the angles vary over the range $103 \cdot 2^{\circ}$ to $121 \cdot 3^{\circ}$, this range corresponding to $60 \sigma$.
Examination of previous recent results for this $\mathrm{Zn}-\mathrm{O}$ distance (Table 3), shows that the mean values are in the range 1.947 (5) to 1.978 (5) $\AA$. The present value is in excellent agreement with these. In addition significant distortion occurs in the $\mathrm{ZnO}_{4}$ tetrahedron in several of these compounds (McDonald \& Cruickshank, 1967; Chao, 1969; Baur \& Tillmanns, 1970) and so the present results are in general agreement with previous ones.

## Phosphate tetrahedron

The individual values of the $\mathrm{P}-\mathrm{O}$ and $\mathrm{O}-\mathrm{O}$ distances are given in Table 2. The weighted mean values are: $\mathrm{P}-\mathrm{O} 1.537$ (3), O-O 2.502 (3) $\AA$. The mean value obtained for the $\mathrm{P}-\mathrm{O}$ bond is in agreement with most of the results of other accurate investigations (Table 3). In this table not all the means and associated error have been given by the authors; some have been calculated from the individual results, in which case the weighted mean has been given.

Table 2. Bond lengths and angles
In this and subsequent tables, atoms in equivalent positions are represented by superscripts

|  | $\begin{aligned} & \text { (i) } \\ & \text { (ii) } \\ & \text { (iii) } \\ & \frac{1}{2} \\ & \frac{1}{2} \end{aligned}$ | $\begin{aligned} \bar{z} ; & \text { (iv) } \\ -z ; & \text { (v) } \\ +z ; & \end{aligned}$ | $\begin{aligned} & \bar{x}, \frac{1}{2}+y, \bar{z} ; \\ & x, \frac{1}{2}-y, z ; \end{aligned}$ | (vi) <br> (vii) | $\begin{aligned} & \frac{1}{2}-x, \bar{y}, \frac{1}{2}+z \\ & \frac{1}{2}+x, y, \frac{1}{2}-z \end{aligned}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{ZnO}_{6}$ octahedron |  |  |  |  |  |  |
| $\mathrm{Zn}(1)-\mathrm{O}(1)$ | 2.099 (8) $\AA$ | $\mathrm{O}(1)-\mathrm{O}(3)$ | 2.864 (9) $\AA$ A |  | $\mathrm{O}(1)-\mathrm{Zn}(1)-\mathrm{O}(3)$ | 84.5 (2) ${ }^{\circ}$ |
| $\mathrm{Zn}(1)-\mathrm{O}(2)$ | 2.116 (9) | $\mathrm{O}(1)-\mathrm{O}(4)$ | 3.033 (9) |  | $\mathrm{O}(1)-\mathrm{Zn}(1)-\mathrm{O}(4)$ | 94.0 (2) |
| $\mathrm{Zn}(1)-\mathrm{O}(3)$ | 2.159 (7) | $\mathrm{O}(2)-\mathrm{O}(3)$ | 3.010 (10) |  | $\mathrm{O}(2)-\mathrm{Zn}(1)-\mathrm{O}(3)$ | 89.5 (2) |
| $\mathrm{Zn}(1)-\mathrm{O}(4)$ | 2.048 (6) | $\mathrm{O}(2)-\mathrm{O}(4)$ | 2.993 (9) |  | $\mathrm{O}(2)-\mathrm{Zn}(1)-\mathrm{O}(4)$ | 91.9 (2) |
| Mean | 2.099 (3) | $\mathrm{O}(3)-\mathrm{O}\left(3^{\prime}\right)$ | 2.943 (9) |  | $\mathrm{O}(3)-\mathrm{Zn}(1)-\mathrm{O}\left(3^{v}\right)$ | 85.9 (2) |
|  |  | $\mathrm{O}(3)-\mathrm{O}(4)$ | 3.057 (9) |  | $\mathrm{O}(4)-\mathrm{Zn}(1)-\mathrm{O}\left(4^{\text {v }}\right.$ ) | $87 \cdot 7$ (2) |
|  |  | $\mathrm{O}(4)-\mathrm{O}\left(4^{\text {V }}\right.$ ) | $2 \cdot 837$ (8) |  |  |  |
|  |  | Mean | $2 \cdot 955$ (3) |  |  |  |
| $\mathrm{ZnO}_{4}$ tetrahedron |  |  |  |  |  |  |
| $\mathrm{Zn}(2)-\mathrm{O}(5)$ | 1.912 (6) Å | $\mathrm{O}(5)-\mathrm{O}\left(6^{\text {dV }}\right.$ ) | $3 \cdot 349$ (8) $\AA$ |  | $\mathrm{O}(5)-\mathrm{Zn}(2)-\mathrm{O}\left(6^{\text {iv }}\right.$ ) | 121.3 (3) ${ }^{\circ}$ |
| $\mathrm{Zn}(2)-\mathrm{O}\left(6^{\text {iV }}\right.$ ) | 1.930 (5) | $\mathrm{O}(5)-\mathrm{O}(7)$ | $3 \cdot 136$ (8) |  | $\mathrm{O}(5)-\mathrm{Zn}(2)-\mathrm{O}(7)$ | $106 \cdot 7$ (2) |
| $\mathrm{Zn}(2)-\mathrm{O}(7)$ | 1.997 (5) | $\mathrm{O}(5)-\mathrm{O}\left(7^{\text {iv }}\right.$ ) | $3 \cdot 102$ (8) |  | $\mathrm{O}(5)-\mathrm{Zn}(2)-\mathrm{O}\left(7^{\text {iv }}\right.$ ) | $105 \cdot 0$ (3) |
| $\mathrm{Zn}(2)-\mathrm{O}\left(7^{\text {lV }}\right.$ ) | 1.999 (5) | $\mathrm{O}\left(6^{\text {iv }}\right)-\mathrm{O}(7)$ | $3 \cdot 193$ (7) |  | $\mathrm{O}\left(6^{\text {iv }}\right)-\mathrm{Zn}(2)-\mathrm{O}(7)$ | 108.8 (3) |
| Mean | $1 \cdot 963$ (3) | $\mathrm{O}\left(6^{\text {iv }}\right)-\mathrm{O}\left(7^{\text {iv }}\right)$ | $3 \cdot 227$ (7) |  | $\mathrm{O}\left(6^{\text {iv }}\right)-\mathrm{Zn}(2)-\mathrm{O}\left(7^{\text {iv }}\right.$ ) | $110 \cdot 4$ (2) |
|  |  | $\mathrm{O}(7)-\mathrm{O}\left(7^{\text {iv }}\right.$ ) | $3 \cdot 131$ (7) |  | $\mathrm{O}(7)-\mathrm{Zn}(2)-\mathrm{O}\left(7^{\text {iv }}\right.$ ) | $103 \cdot 2$ (2) |
|  |  | Mean | $3 \cdot 189$ (3) |  |  |  |
| Phosphate tetrahedron |  |  |  |  |  |  |
| $\mathrm{P}-\mathrm{O}(4)$ | 1.521 (6) $\AA$ | $\mathrm{O}(4)-\mathrm{O}(5)$ | 2.504 (8) $\AA$ |  | $\mathrm{O}(4)-\mathrm{P}-\mathrm{O}(5)$ | $111 \cdot 1(3)^{\circ}$ |
| $\mathrm{P}-\mathrm{O}(5)$ | 1.515 (6) | $\mathrm{O}(4)-\mathrm{O}(6)$ | 2.484 (8) |  | $\mathrm{O}(4)-\mathrm{P}-\mathrm{O}(6)$ | $108 \cdot 7$ (3) |
| $\mathrm{P}-\mathrm{O}(6)$ | 1.536 (5) | $\mathrm{O}(4)-\mathrm{O}(7)$ | 2.526 (8) |  | $\mathrm{O}(4)-\mathrm{P}-\mathrm{O}(7)$ | $109 \cdot 9$ (3) |
| $\mathrm{P}-\mathrm{O}(7)$ | $1 \cdot 564$ (5) | $\mathrm{O}(5)-\mathrm{O}(6)$ | 2.534 (8) |  | $\mathrm{O}(5)-\mathrm{P}-\mathrm{O}(6)$ | $112 \cdot 3$ (4) |
| Mean | $1 \cdot 537$ (3) | $\mathrm{O}(5)-\mathrm{O}(7)$ | 2.513 (8) |  | $\mathrm{O}(5)-\mathrm{P}-\mathrm{O}(7)$ | 109.4 (3) |
|  |  | $\mathrm{O}(6)-\mathrm{O}(7)$ | 2.465 (7) |  | $\mathrm{O}(6)-\mathrm{P}-\mathrm{O}(7)$ | $105 \cdot 3$ (3) |
|  |  | Mean | $2 \cdot 502$ (3) |  |  |  |

The only result significantly different from the present one is that from one of the phosphate groups in $\mathrm{Ca}\left(\mathrm{H}_{2} \mathrm{PO}_{4}\right)_{2} . \mathrm{H}_{2} \mathrm{O}$ (Dickens \& Bowen, 1971). An examination of the individual bond lengths in this structure gives an average of $1.535 \AA$ but the weighted mean (calculated by the present writer from the individual values) is $1.512 \AA$; this large difference is due to the better accuracy of the shorter bonds. However in this particular case the difference may be accentuated by rounding off the individual errors in the original paper to one significant figure which is either 1 or 2. As a result it is suggested that authors should either not round off errors to one significant figure or report
a weighted mean calculated from data prior to rounding off.

Examination of the individual distances and angles in Table 2 indicates that the tetrahedron is distorted both in bond lengths and bond angles. In some cases the differences from the means are greater than three times the appropriate standard deviation. This is particularly so for $\mathrm{O}(7)$ for which the $\mathrm{P}-\mathrm{O}$ bond length is large enough to suggest the possibility of a $\mathrm{P}-\mathrm{OH}$ bond. However, on the assumption that the hydrogen bond would be directed away from the P , this would put the H atom near to $\mathrm{Zn}(2)$. Hence this distortion must be due to some other cause.

Table 3. Comparison of observed bond lengths


Examination of the coordination about each O atom shows that for the three atoms $O(4), O(5)$ and $O(6)$, each is involved in the coordination polyhedron about one Zn whereas $\mathrm{O}(7)$ is involved in the coordination about two Zn atoms. In parahopeite (Chao, 1969) there is a similar state of affairs; again there is one O atom involved in coordination about two Zn atoms whereas the other three are coordinated to a single Zn , and again the first atom has a significantly longer $\mathrm{P}-\mathrm{O}$ bond than the others.

Significant distortion of the orthophosphate tetrahedron that may be accounted for in the same manner also occurs in $\mathrm{Mn}_{2}\left(\mathrm{PO}_{4}\right) \mathrm{Cl}$ (Rea \& Kostiner, 1972a), $\mathrm{Mg}_{2}\left(\mathrm{PO}_{4}\right) \mathrm{Cl}$ (Rea \& Kostiner, 1972c) $\mathrm{NaAl}_{3}\left(\mathrm{PO}_{4}\right)_{2}(\mathrm{OH})_{4}$ (Gatehouse \& Miskin, 1974).
The case of $\mathrm{Mn}_{2}\left(\mathrm{PO}_{4}\right) \mathrm{Cl}$ (Rea \& Kostiner, 1972a) is interesting in that one oxygen is involved in three coordination polyhedra while the other three oxygens are involved in two coordination polyhedra and the first oxygen is involved in a significantly longer $\mathrm{P}-\mathrm{O}$ bond. However, in the chemically similar $\mathrm{Mn}_{2}\left(\mathrm{PO}_{4}\right) \mathrm{F}$ (Rea \& Kostiner, 1972b) all O atoms are involved in two coordination polyhedra and the linear distortion of the phosphate group is not significant.

A similar effect occurs in the orthoarsenates (Calvo \& Leung, 1969) but the magnitude is much greater; Finney (1966) has found individual As-O bonds in the range 1.615 to $1.773 \AA$ in $\mathrm{Cu}_{2}\left(\mathrm{AsO}_{4}\right) \mathrm{OH} .3 \mathrm{H}_{2} \mathrm{O}$. The greater effect is no doubt due to the As-O bond being longer and thus weaker than the $\mathrm{P}-\mathrm{O}$ bond and because of this the effect of being involved in coordination to a cation is somewhat greater.

There are, of course, other causes of distortion in the orthophosphate tetrahedron. In the case of $\mathrm{Na}_{3} \mathrm{PO}_{3}(\mathrm{NaOH})_{0-0.25} .12 \mathrm{H}_{2} \mathrm{O}$ it was found (Tillmanns \& Baur, 1971) that if the O atoms of a phosphate group are receptor atoms of a hydrogen bond then this also lengthens the appropriate $\mathrm{P}-\mathrm{O}$ bond. In the present case three of the four phosphate oxygens are receptor atoms for hydrogen bonds and these three are associated with the three shortest P-O bonds. Hence this effect is much smaller than that of being coordinated to an extra cation as the fourth oxygen, this one would expect from a consideration of the interatomic distances involved.

Consider the sum of the angles about each of the phosphate oxygens in turn; for $\mathrm{O}(7)$ the nearest neighbours are two Zn and a P atom, for $\mathrm{O}(5)$ and $\mathrm{O}(6)$ they are a Zn, a P and a donor O atom, while in addition to these $O(4)$ may have a second donor. The sum of the
angles about $\mathrm{O}(5), \mathrm{O}(6)$ and $\mathrm{O}(7)$ are $325 \cdot 0$ (6), $358.6(5)^{\circ}$ and $360.6(5)^{\circ}$ respectively while if we ignore the effect of the second donor atom this sum is $359.9(5)^{\circ}$ for $\mathrm{O}(4)$. In view of the fact that this is not significantly different from the others, it would appear that the hydrogen-bonding effect of $\mathrm{H}(24)$ is too weak to affect the configuration.

## Hydrogen bonding

As already mentioned the $\mathrm{H}_{2} \mathrm{O}$ molecules were confirmed by a difference synthesis. The positions of three H atoms $\mathrm{H}(14), \mathrm{H}(35)$ and $\mathrm{H}(36)$ were clearly obtained but that associated with $\mathrm{O}(2)$ could be directed either approximately towards $\mathrm{O}(3)$, i.e. $\mathrm{H}(23)$ or towards $\mathrm{O}(4)$, i.e. $\mathrm{H}(24)$. In an attempt to decide which of these could be the fourth H atom the distances and angles involved in both schemes were calculated and so were those involving $\mathrm{H}(14), \mathrm{H}(35)$, and $\mathrm{H}(36)$ for comparison (Table 4).

Regardless of whether $\mathrm{H}(23)$ or $\mathrm{H}(24)$ is considered to be correct, the distance and angles involving them are similar with one exception, $\mathrm{O}_{a}-\mathrm{O}_{b}$. By itself this might be taken as evidence that $\mathrm{H}(23)$ is the more probable as the value in this case, $2 \cdot 926$ (10) $\AA$, lies well within the accepted range of hydrogen-bond lengths (Pimentel \& McClellan, 1960), whereas the same interatomic distance for $H(24), 3 \cdot 123$ (9) $\AA$, is near the end of the range. This view is reinforced by the fact that the shortest non-bonded $\mathrm{O}-\mathrm{O}$ distance is $3.077 \AA$ and so if $\mathrm{H}(24)$ is present the bond must be weak or non-bonding.

On the other hand one would expect a receptor atom to be an oxygen rather than a water molecule as the negative charge on the former would be larger than the latter due to the effect of the hydrogens. This in turn would lead to a greater dipole in the bond.

However, previous discussion concluded that neither $\mathrm{H}(23)$ nor $\mathrm{H}(24)$ took part in hydrogen bonding and the most probable bonding is that shown schematically in Fig. 4. Thus the only evidence would appear to be computational in that $\mathrm{H}(24)$ partially refined whereas $\mathrm{H}(23)$ did not; however in this case we are left with the query as to why it did not refine fully as the other three hydrogens did. This raises another possibility: could the hydrogen be in positions $\mathrm{H}(23)$ and $\mathrm{H}(24)$ at random? The angles $\mathrm{H}(23)-\mathrm{O}(2)-\mathrm{H}\left(23^{v}\right)\left(143 \pm 18^{\circ}\right)$ and $\mathrm{H}(24)-\mathrm{O}(2)-\mathrm{H}\left(24^{v}\right)\left(100 \pm 14^{\circ}\right)$ are not significantly different from the tetrahedral angle and neither is $\mathrm{H}(23)-\mathrm{O}(2)-\mathrm{H}\left(24^{\vee}\right)\left(144 \pm 16^{\circ}\right)$, thus allowing com-

Table 4. Distances $(\AA)$ and angles $\left({ }^{\circ}\right)$ involved in hydrogen bonding

| Donor |  | Receptor |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{O}_{0}$ (water) | H | $\mathrm{O}_{\text {b }}$ | $\mathrm{O}_{a}-\mathrm{O}_{b}$ | $\mathrm{O}_{a}-\mathrm{H}$ | $\mathrm{H}-\mathrm{O}^{\text {b }}$ | $\mathrm{O}_{b}-\mathrm{O}_{a}-\mathrm{O}_{b}$ | $\mathrm{H}-\mathrm{O}_{a}-\mathrm{H}$ | $\mathrm{O}_{a}-\mathrm{H}-\mathrm{O}_{b}$ |
| $\mathrm{O}(1)$ | H(14) | $\mathrm{O}(4)$ | 2.794 (9) | 1.33 (22) | 1.73 (23) | 61.0 (2) | 116 (14) | 131 (16) |
| $\mathrm{O}(2)$ | H(23) | $\mathrm{O}(3)$ | 2.926 (10) | 1.06 (21) | 2.26 (24) | $60 \cdot 4$ (3) | 143 (18) | 119 (16) |
| $\mathrm{O}(2)$ | H(24) | $\mathrm{O}(4)$ | $3 \cdot 123$ (9) | 1.24 (21) | 2.05 (22) | 54.0 (2) | 100 (14) | 142 (15) |
| $\mathrm{O}(3)$ | H(35) | $\mathrm{O}(5)$ | 3.029 (9) | 1.35 (23) | 1.95 (23) | $92 \cdot 0$ (2) | 114 (15) | $\{132$ (17) |
| $\mathrm{O}(3)$ | H(36) | O(6) | $2 \cdot 708$ (8) | $1 \cdot 18$ (23) | 1.54 (23) | 92.0 (2) | 114 (15) | 166 (20) |

plete randomness among the sites. In some ways this is the most satisfactory explanation as it accounts for both the appearance of the difference map and the difficulty in refining the atoms.

Although in the cases of the hydrogen bonds which are known to exist, the angles subtended by the hydrogen at the water oxygen are tetrahedral it is noticeable that in one case, $\mathrm{O}(1)$, the angle subtended by the receptor oxygens is considerably smaller. Although this effect has been obtained previously in oxalic acid dihydrate (Garrett, 1954), by neutron diffraction, the present angle is smaller than that found before. In the case of oxalic acid dihydrate it was found that the hydrogen bond was non-collinear so that the H atoms subtended an angle of $109^{\circ}$ at the donor oxygen although the receptor atoms did not. This may be the case in hopeite but the errors in the angles associated with the hydrogen atoms are too large to confirm this.

## Analysis of thermal vibrations

The r.m.s. displacements and the direction cosines of the thermal vibration ellipsoids are given in Table 5. The errors in the displacements were obtained by calculating the vibration ellipsoids for the values in Table 1 plus the standard deviations, and minus the standard deviations and averaging over similar atoms.
In the case of the $\mathrm{ZnO}_{6}$ octahedron it can be seen that for $\mathrm{Zn}(1)$ and $\mathrm{O}(1)$ the vibration ellipsoids approximate to a prolate sphere with the unique axis of vibration parallel to the $b$ axis while for $\mathrm{O}(3)$ the distortion from spherical symmetry is not significant and for $\mathrm{O}(4)$ the vibration ellipsoid again approximates to a prolate sphere. Considering these O atoms about


Fig. 4. Schematic diagram of proposed bonding.

Table 5. R.m.s. displacements and orientation with respect to crystallographic axes

| Zn(1) | R.m.s. displacement | $\underset{a}{\text { Direction cosine with respect to }}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $0 \cdot 157$ (2) | 0.000 | 1.000 | 0.000 |
|  | $0 \cdot 122$ (2) | $0 \cdot 212$ | 0.000 | 0.977 |
| $\mathrm{Zn}(2)$ | $0 \cdot 110$ (2) | $0 \cdot 977$ | 0.000 | -0.212 |
|  | $0 \cdot 109$ (2) | $0 \cdot 000$ | 0.999 | -0.051 |
|  | 0.071 (2) | 0.021 | 0.051 | 0.998 |
|  | 0.066 (2) | 0.999 | -0.001 | -0.021 |
| P | 0.088 (4) | 0.807 | 0.589 | -0.031 |
|  | 0.085 (4) | 0.573 | -0.771 | 0.276 |
|  | 0.073 (4) | -0.138 | 0.241 | 0.961 |
| $\mathrm{O}(1)$ | $0 \cdot 180$ (14) | 0.000 | -1.000 | 0.000 |
|  | $0 \cdot 116$ (14) | $0 \cdot 863$ | 0.000 | -0.505 |
|  | $0 \cdot 092$ (14) | -0.505 | 0.000 | -0.863 |
| $\mathrm{O}(2)$ | $0 \cdot 204$ (14) | 0.000 | -1.000 | $0 \cdot 000$ |
|  | $0 \cdot 143$ (14) | $0 \cdot 169$ | 0.000 | -0.986 |
|  | 0.069 (14) | 0.986 | 0.000 | $0 \cdot 169$ |
| $\mathrm{O}(3)$ | $0 \cdot 169$ (14) | -0.305 | -0.683 | -0.664 |
|  | $0 \cdot 151$ (14) | -0.243 | -0.618 | $0 \cdot 748$ |
|  | $0 \cdot 123$ (14) | $0 \cdot 921$ | 0.390 | -0.023 |
| $\mathrm{O}(4)$ | $0 \cdot 161$ (14) | $0 \cdot 915$ | -0.232 | 0.331 |
|  | $0 \cdot 115$ (14) | 0.318 | 0.919 | $-0.235$ |
|  | $0 \cdot 105$ (14) | -0.249 | $0 \cdot 320$ | 0.914 |
| O(5) | $0 \cdot 221$ (14) | 0.975 | $0 \cdot 211$ | -0.074 |
|  | 0.129 (14) | 0.215 | $-0.975$ | 0.051 |
|  | 0.056 (14) | 0.061 | 0.066 | 0.996 |
| $\mathrm{O}(6)$ | $0 \cdot 167$ (14) | -0.042 | 0.192 | 0.980 |
|  | $0 \cdot 113$ (14) | -0.108 | -0.977 | $0 \cdot 187$ |
|  | 0.054 (14) | 0.993 | -0.098 | 0.062 |
| O(7) | $0 \cdot 127$ (14) | -0.252 | -0.914 | -0.317 |
|  | 0.098 (14) | $0 \cdot 344$ | -0.390 | $0 \cdot 854$ |
|  | $0 \cdot 064$ (14) | $0 \cdot 904$ | -0.106 | -0.413 |

$\mathrm{Zn}(1)-\mathrm{O}(1), \mathrm{O}(2), \mathrm{O}(3)$ and $\mathrm{O}(4)$ - the shortest axes of vibration make angles of $10^{\circ}, 31^{\circ}, 73^{\circ}$ and $68^{\circ}$ with the respective bond. In view of the shape of the vibration ellipsoids of $\mathrm{O}(3)$ and $\mathrm{O}(4)$, the errors in the last two angles are probably very large and probably what are more important are the directions of the largest axes. These make angles of $90^{\circ}, 90^{\circ}, 75^{\circ}$ and $83^{\circ}$ with the respective bond for $\mathrm{O}(1), \mathrm{O}(2), \mathrm{O}(3)$ and $\mathrm{O}(4)$.

Consider $\mathrm{Zn}(2)$; the vibration ellipsoid for this approximates to a prolate sphere with the unique axis parallel to the $b$ axis. However, there would appear to be nothing special about the four oxygen neighbours, $\mathrm{O}(5), \mathrm{O}\left(6^{\mathrm{iv}}\right), \mathrm{O}(7)$ and $\mathrm{O}\left(7^{\mathrm{iv}}\right)$. Calculation of the orientations of the vibration ellipsoids of these four atoms indicates that the longest axes are at angles of $84^{\circ}, 87^{\circ}$, $82^{\circ}$ and $76^{\circ}$ with respect to the respective bond while the shortest axes are at angles of $52^{\circ}, 56^{\circ}, 54^{\circ}$ and $15^{\circ}$ to the respective bonds for $\mathrm{O}(5), \mathrm{O}\left(6^{\mathrm{iv}}\right), \mathrm{O}(7)$ and $\mathrm{O}\left(7^{\mathrm{iv}}\right)$ respectively.

The vibration of the phosphorus atom is isotropic, while of its neighbours, only the vibration ellipsoid of $\mathrm{O}(4)$ would appear to be anything other than general; it approximates to a prolate sphere. In the case of the four oxygen neighbours, the longest axes of vibration of $\mathrm{O}(4), \mathrm{O}(5), \mathrm{O}(6)$ and $\mathrm{O}(7)$ are at angles of $48^{\circ}, 89^{\circ}$, $70^{\circ}$, and $74^{\circ}$ to the respective P-O bond, while the shortest ones are at angles of $65^{\circ}, 7^{\circ}, 30^{\circ}$ and $62^{\circ}$ respectively.

## Bond lengths corrected for thermal vibrations

Busing \& Levy (1964) have pointed out that one of the effects of thermal vibration is to alter the apparent bond lengths and they showed how to correct for the effect. Values of the bond lengths are included in Table 6, corrected for three of the cases they discuss; $(A)$ the atoms are vibrating in phase, $(B)$ the outer atoms are 'riding' on the inner ones and $(C)$ the atoms are moving independently of each other.

Table 6. Bond lengths corrected for thermal vibrations

|  | Uncorrected bond length | Corrected bond length |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $A$ (in | $B$ | $C$ (inde- | dard |
|  |  | phase) | (riding) | pendent) |  |
| $\mathrm{Zn}(1)-\mathrm{O}(1)$ | 2.099 A | 2.099 A | $2 \cdot 101 \AA$ | 2.118 A | $0.008 \AA$ |
| $\mathrm{Zn}(1)-\mathrm{O}(2)$ | $2 \cdot 116$ | $2 \cdot 117$ | $2 \cdot 121$ | $2 \cdot 137$ | 0.009 |
| $\mathrm{Zn}(1)-\mathrm{O}(3)$ | $2 \cdot 159$ | 2.159 | $2 \cdot 161$ | 2.177 | 0.007 |
| $\mathrm{Zn}(1)-\mathrm{O}(4)$ | 2.048 | 2.048 | $2 \cdot 049$ | 2.065 | 0.006 |
| Mean | 2.099 | 2.099 | 2.101 | $2 \cdot 118$ | 0.003 |
| $\mathrm{Zn}(2)-\mathrm{O}(5)$ | 1.912 | 1.916 | 1.923 | 1.930 | 0.006 |
| $\mathrm{Zn}(2)-\mathrm{O}\left(6^{\text {iv }}\right.$ ) | ) 1.930 | 1.931 | 1.936 | 1.942 | 0.005 |
| $\mathrm{Zn}(2)-\mathrm{O}(7)$ | 1.996 | 1.996 | 1.998 | 2.005 | 0.005 |
| $\mathrm{Zn}(2)-\mathrm{O}\left(7^{\text {iv }}\right)$ | ) 1.999 | 1.999 | 2.002 | 2.009 | 0.005 |
| Mean | 1.963 | 1.964 | 1.968 | 1.975 | 0.003 |
| $\mathrm{P}-\mathrm{O}(4)$ | 1.521 | 1.523 | 1.529 | 1.538 | 0.006 |
| $\mathrm{P}-\mathrm{O}(5)$ | 1.515 | 1.521 | 1.532 | 1.542 | 0.006 |
| $\mathrm{P}-\mathrm{O}(6)$ | 1.536 | 1.538 | 1.544 | 1.552 | 0.005 |
| $\mathrm{P}-\mathrm{O}(7)$ | 1.564 | $1 \cdot 565$ | 1.567 | 1.575 | 0.005 |
| Mean | 1.537 | 1.539 | 1.545 | 1.554 | 0.003 |

In the case of the oxygen octahedra about $\mathrm{Zn}(1)$ the longest r.m.s. displacements of the O atoms are almost perpendicular to the respective bonds while the displacement along the bond is much shorter. In addition this displacement is approximately the same as for $\mathrm{Zn}(1)$. This suggests that these O atoms are riding on the zinc; however $\mathrm{O}(4)$ is also bonded to phosphorus and so its vibration ellipsoid must be affected by both $\mathrm{Zn}(1)$ and P , suggesting that it may be vibrating with some independence of both. However, it would appear that, in general, the riding corrected value would be the best, thus $\mathrm{Zn}-\mathrm{O}=2 \cdot 101$ (3) $\AA$ when the zinc is octahedrally coordinated.

In the case of the water tetrahedron about $\mathrm{Zn}(2)$, the longest r.m.s. displacements of the O atoms are almost perpendicular to the respective bonds while the displacement along the bond is shorter. In addition the displacement is approximately the same as for the zinc and so the riding model would appear preferable. However, the picture is more complicated; all four O atoms are bonded to a P atom and one of them $[\mathrm{O}(7)]$ to another $\mathrm{Zn}(2)$ as well. Probably as a result of these other bonds, only for one atom $\left[\mathrm{O}\left(7^{\mathrm{iv}}\right)\right]$ is the shortest axis of vibration even approximately parallel to the respective $\mathrm{Zn}-\mathrm{O}$ bond. Another way of examining this is to look at the phosphate tetrahedron. One O atom, $\mathrm{O}(4)$, is vibrating independently of the P atom and appears to be riding on $\mathrm{Zn}(1)$. For the other three
oxygens, however, their longest r.m.s. displacements are almost perpendicular to the respective bonds while the displacement along the bond is shorter. Again this displacement is approximately the same as for the phosphorus and so the riding model would again appear preferable. However these three O atoms cannot be riding simultaneously on $\mathrm{Zn}(2)$ and P and thus it would appear that oxygen atoms coordinated to both $\mathrm{Zn}(2)$ and P must, to a considerable extent, be vibrating independently of both. Consequently it is not possible to evaluate a corrected bond length for either the tetrahedral $\mathrm{Zn}-\mathrm{O}$ or the $\mathrm{P}-\mathrm{O}$ distance, but presumably the best values would be between those for the riding and independent models.

The author would like to thank Dr P. Pauling for facilities for collecting the data and Dr R. W. Baker for assistance in collecting the data and computing.

## References

Ansell, G. B. \& Katz, L. (1966). Acta Cryst. 21, 482-485.
Araki, T. \& Zoltai, T. (1968). Z. Kristallogr. 127, 21-33.
bacon, G. E. \& Curry, N. A. (1962). Proc. Roy. Soc. (Lond.) A 266, 95-108.
Baur, W. H. (1964). Acta Cryst. 17, 863-864.
Baur, W. H. \& Tillmanns, E. (1970). Z. Kristallogr. 131, 213-221.
Bоoth, A. D. (1948). Fourier Technique in X-ray Organic Structure Analysis, pp. 62-65. Cambridge Univ. Press.
Busing, W. R. \& Levy, H. A. (1964). Acta Cryst. 17, 142146.

Calvo, C. \& Leung, K. Y. (1969). Z. Kristallogr. 130, 231-233.
Catti, M. \& Ferraris, G. (1974). Acta Cryst. B30, 1-6.
Chao, G. Y. (1969). Z. Kristallogr. 130, 261-266.
Dickens, B. \& Bowen, J. S. (1971). Acta Cryst. B27, 2247-2255.
Dickens, B., Bowen, J. S. \& Brown, W. E. (1972). Acta Cryst. B28, 797-806.
Dickens, B., Brown, W. E., Kruger, G. J. \& Stewart, J. M. (1973). Acta Cryst. B29, 2046-2056.

Dickens, B., Prince, E., Schroeder, L. W. \& Brown, W. E. (1973). Acta Cryst. B29, 2057-2070.

Ferraris, G. \& Franchini-Angela, M. (1974). Acta Cryst. B30, 504-510.
Finney, J. J. (1966). Acta Cryst. 21, 437-440.
Gamidov, R. S., Golovachev, V. P., Mamedov, Kh. S. \& Belov, N. V. (1963). Dokl. Akad. Nauk SSSR, 150, 106-109.
Garrett, B. S. (1954). Oak Ridge National Laboratory Report No. 1745; quoted in Bacon \& Curry (1962).
Gatehouse, b. M. \& Miskin, B. K. (1974). Acta Cryst. B30, 1311-1317.
Greenblatt, M., Banks, E. \& Post, B. (1969). Acta Cryst. B25, 2170-2171.
International Tables for X-ray Crystallography (1962). Vol. III, p. 201. Birmingham: Kynoch Press.
Khan, A. A., Roux, J. P. \& James, W. J. (1972). Acta Cryst. B28, 2065-2069.
Kleber, W. Liebau, F. \& Piatkowiak, E. (1961). Acta Cryst. 14, 795.
Kumbasar, I. \& Finney, J. J. (1968). Miner. Mag. 36, 621-624.

Liebau, F., (1965). Acta Cryst. 18, 352-354.
Liminga, R. (1965). Acta Chem. Scand. 19, 1629-1642.
Liminga, R. (1966). Acta Chem. Scand. 20, 2483-2496.
McDonald, W. S. \& Cruickshank, D. W. J. (1967). Z. Kristallogr. 124, 180-191.
Mackie, P. E., Elliott, J. C. \& Young, R. A. (1972). Acta Cryst. B28, 1840-1848.
Mamedov, Kh. S., Gamidov, R. \& Belov, N. V. (1961). Sov. Phys. Crystallogr. 5, 91-94.
Mason, R. (1964). Acta Cryst. 17, 547-555.
Palache, C., Berman, H. \& Frondel, C. (1963). Dana's System of Mineralogy, 7th ed. Vol. II, p. 735.
Pimentel, G. C. \& McClellan, A. L. (1960). The Hydrogen Bond, p. 280. San Francisco, London: Freeman.
Rea, J. R. \& Kostiner, E. (1972a). Acta Cryst. B28, $2504-$ 2509.

Rea, J. R. \& Kostiner, E. (1972b). Acta Cryst. B28, 25252529.

Rea, J. R. \& Kostiner, E. (1972c). Acta Cryst. B28, 34613464.

Schulten, M. A. de (1904). Bull. Soc. Fr. Minér. 27, 100103.

Spencer, L. J., (1908). Miner. Mag. 15, 1-38.
Stählin, W. \& Oswald, H. R. (1970). Acta Cryst. B26, 860-863.
Sudarsanan, K. \& Young, R. A. (1972). Acta Cryst. B28, 3668-3670.
Sudarsanan, K. \& Young, R. A. (1974). Acta Cryst. B30, 1381-1386.
Sudarsanan, K., Young, R. A. \& Donnay, J. D. H. (1973a). Acta Cryst. B 29, 808-814.
Sudarsanan, K., Young, R. A. \& Donnay, J. D. H. (1973b). Acta Cryst. B29, 2649.
Sutor, D. J. (1967). Acta Cryst. 23, 418-422.
Tillmanns, E. \& Baur, W. H. (1971). Acta Cryst. B27, 2124-2132.
Whitaker, A. (1973). Z. Kristallogr. 137, 194-219.
Whitaker, A. \& Jeffery, J. W. (1970). Acta Cryst. B26, 1429-1440.
Wolfe, C. W. (1940). Amer. Min. 25, 787-809.

# The Crystal Structure and Molecular Conformation of Cyclo-L-prolyl-L-prolyl-L-hydroxyproline, a Cyclic Tripeptide 

By G. Kartha and G. Ambady<br>Center for Crystallographic Research, Roswell Park Memorial Institute, 666 Elm Street, Buffalo, New York 14203, U.S.A.

(Received 8 November 1974;-accepted 24 February 1975)


#### Abstract

The cyclization of three peptides to form a nine-membered ring introduces a certain amount of strain in the geometry of the peptide unit. This first crystal structure analysis of a cyclic tripeptide $\left(\mathrm{C}_{15} \mathrm{H}_{21} \mathrm{O}_{4} \mathrm{~N}_{3}\right)$ has shown that all three peptide units are in the cis conformation. The planarity of the peptide units is not unduly disturbed, but the three bonds involving the nitrogen atom are not coplanar. The three pyrrolidine rings are puckered differently, the atoms out of plane in the three cases being $\mathrm{N}, \mathrm{C}^{\alpha}$ and $\mathrm{C}^{\beta}$ respectively. The hydroxyl oxygen of the hypro group in this molecule is disordered, occupying two positions, both of which are able to participate in hydrogen bonding. The crystal belongs to the orthorhombic space group $P 2_{1} 2_{1} 2_{1}$ with unit-cell parameters, $a=16 \cdot 284, b=9 \cdot 449, c=9 \cdot 260 \AA$ and $Z=4$. The structure was solved by direct methods with the MULTAN program and refined by full-matrix least-squares calculations to a reliability index of $6 \cdot 2 \%$.


## Introduction

In proteins and polypeptides, the building block is almost invariably the trans peptide unit. However there are situations where the cis peptide is incorporated in molecules for specific stereochemical reasons. A class of compounds containing cis peptides comprises the cyclic peptides having a small number of peptide units. In the diketopiperazines, the two cis peptides are cyclized to form a six-membered ring (Corey, 1938; Karle, 1972). It has been predicted that three peptides can be cyclized only if all three are in the cis conformation (Venkatachalam, 1968). As the number of peptide units increases, the trans peptide is increasingly preferred (Dale \& Titlestad, 1970; Kopple, Onishi \& Go,

1969; Kopple, 1971). Cyclo-L-Pro-l-Pro-L-Pro is the first cyclic tripeptide to be synthesized (Rothe, Steffan \& Rothe, 1965). The derivatives of this compound were later synthesized and studied by n.m.r. spectroscopy by Blout and coworkers (Debar, Torchia \& Blout, 1971). The structural parameters of cyclo-L-Pro-L-Pro-L-Pro were deduced from empirical calculations using stereochemical criteria (Venkatachalam, 1968). In these calculations it was shown that the three peptides have to be in the cis conformation for ring closure to occur. The two assumptions made in these calculations were: (1) the proline rings were planar and (2) the three bonds connected to the nitrogen atom were coplanar. To avoid short intramolecular contacts between the $\mathrm{C}^{\alpha}$ protons, the torsion angle $\omega$ (a measure


[^0]:    * A list of structure factors has been deposited with the British Library Lending Division as Supplementary Publication No. SUP 30961 ( 9 pp.). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 13 White Friars, Chester CH1 1NZ, England.

